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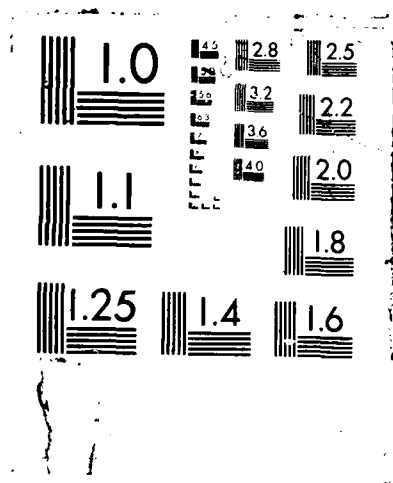
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PARAMETRIC STUDY OF ACCELERATION-INDUCED STRESSES  
AND FREQUENCY CHANGES IN DOUBLY-ROTATED CRYSTAL RESONATORS

Final Report

July 16, 1984 - September 30, 1987

ARO Contract No. DAAG29-84-K-0133

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER <i>ARL 21353.7-EG</i>	2. GOVT ACCESSION NO. N/A	3. RECIPIENT'S CATALOG NUMBER N/A
4. TITLE (and Subtitle) PARAMETRIC STUDY OF ACCELERATION-INDUCED STRESSES AND FREQUENCY CHANGES IN DOUBLY ROTATED CRYSTAL RESONATORS		5. TYPE OF REPORT & PERIOD COVERED Final Report 7/16/84 - 9/30/87
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) P. C. Y. Lee		8. CONTRACT OR GRANT NUMBER(s) DAAG29-84-K-0133
9. PERFORMING ORGANIZATION NAME AND ADDRESS Dept of Civil Engineering & Operations Research Princeton University, Princeton, NJ 08544		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 21353-EG
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE November 15, 1987
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)  NA		
18. SUPPLEMENTARY NOTES  The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Acceleration sensitivity of crystal resonators, Doubly rotated cuts of quartz disks, Thickness-shear resonances, Effects of plate thickness, Support configurations, Support structures.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  (See reverse side)		

The objective of the research is to predict the acceleration sensitivity, i.e., the changes of resonance frequencies in crystal resonators due to applied, steady accelerations, and to identify the effects on the acceleration sensitivity from various parameters, such as the direction of acceleration, crystal plate orientation, plate thickness, support configuration and support structures.

The ratios of the frequency shift to the thickness-shear resonance frequency are in the range of  $10^{-8}$  to  $10^{-10}$ . These frequency changes, although very small in numerical values, are regarded very significant in applications in ultra-precision frequency control.

The complexity of the problem mainly comes from the constitutive equations of the crystal: (a) For doubly-rotated cuts of quartz resonators, the stress-strain relations possess triclinic symmetry, the most general anisotropic relations; (b) For accurate prediction of frequency changes, the third-order elastic stiffness coefficients, the nonlinear terms, must be taken into account.

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PARAMETRIC STUDY OF ACCELERATION-INDUCED STRESSES  
AND FREQUENCY CHANGES IN DOUBLY-ROTATED CRYSTAL RESONATORS

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I. STATEMENT OF THE PROBLEM STUDIED

The objective of the research is to predict the acceleration sensitivity, i.e., the changes of resonance frequencies in crystal resonators due to applied, steady accelerations, and to identify the effects on the acceleration sensitivity from various parameters, such as the direction of acceleration, crystal plate orientation, plate thickness, support configuration and support structures.

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## II. SUMMARY OF THE MOST IMPORTANT RESULTS

1. A numerical-analytical approach has been developed for studying force- and acceleration-sensitivities of crystal resonators.

In this approach, complex initial fields of stress and deformation are obtained from the numerical solutions of six coupled equations of equilibrium for doubly-rotated crystal resonators based on the finite element method (see the third paper listed in Section III as III<sub>3</sub>). The frequency equation for thickness vibrations is obtained from the two-dimensional equations of motion of incremental vibrations. This frequency equation, in which the initial strain and initial deformation tensors appear as given spatial functions, is solved analytically by a perturbation method.

This numerical-analytical approach is employed to predict force sensitivity of circular crystal resonators subject to a pair of diametrical forces. The predicted results are compared with experimental data with close agreement for all of the eight doubly rotated cuts (see III<sub>6</sub>).

This same approach is further employed to predict acceleration sensitivity of circular SC-cut quartz resonators. The effects of plate thickness, support configurations, and support structures on the acceleration sensitivity are studied systematically. It is found that the sensitivity can be reduced by carefully controlling these parameters (see III<sub>7</sub>).

2. Two-dimensional equations of motion for piezoelectric crystal plates are derived from the three-dimensional equations of linear piezoelectricity. These equations are shown to provide accurate predictions of thickness

resonances and can accommodate electric boundary conditions for plates with electroded or unelectroded faces (see III<sub>5</sub>).

These equations are employed to calculate the resonance frequencies of energy trapped thickness vibrations in rectangular or circular piezoelectric crystal plates with electroded or unelectroded faces. The predicted results are compared with existing analytical results and experimental data with good agreement (see III<sub>8</sub>).



### III. LIST OF PUBLICATIONS AND TECHNICAL REPORTS

1. P. C. Y. Lee and J. P. Hou, "Vibrations of doubly rotated piezoelectric crystal strips with a pair of electrode-plated, traction-free edges," *Proc. 39th Ann. Freq. Control Symp.*, pp. 453-461, 1985.
2. Y. K. Yong and P. C. Y. Lee, "Frequency-temperature behavior of flexural and thickness-shear vibrations of rectangular, rotated Y-cut quartz plates," *Proc. 39th Ann. Freq. Control Symp.*, pp. 415-426, 1985.
3. P. C. Y. Lee and M. S. H. Tang, "Initial stress field and resonance frequencies of incremental vibrations in crystal resonators by finite element method," *Proc. 40th Ann. Freq. Control Symp.*, pp. 152-160, 1986.
4. P. C. Y. Lee and Y. K. Yong, "Frequency-temperature behavior of thickness vibrations of doubly rotated quartz plates affected by plate dimensions and orientations," *J. Appl. Phys.*, 60(7), pp. 2327-2342, 1986.
5. P. C. Y. Lee, S. Syngellakis and J. P. Hou, "A two-dimensional theory for high-frequency vibrations of piezoelectric crystal plates with or without electrodes," *J. Appl. Phys.*, 61(4), pp. 1249-1262, 1987. Also in *Proc. IEEE 1986 Ultrasonics Symp.*, pp. 349-354.
6. P. C. Y. Lee and M. S. H. Tang, "Thickness vibrations of doubly rotated crystal plates under initial deformations," to be published in *IEEE Trans. on Ultrasonics, Ferroelectrics and Freq. Control*, Nov., 1987.
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8. P. C. Y. Lee, X. Guo and M. S. H. Tang, "Coupled and energy trapped thickness vibrations in piezoelectric crystal plates," to be published in *Proc. IEEE 1987 Ultrasonics Symp.*

IV. LIST OF PARTICIPATING SCIENTIFIC PERSONNEL

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Mr. M. S. H. Tang (expecting to obtain his Ph.D. degree within  
3 months)

Mr. A. M. A. Mohammad, Post-general graduate assistant

Mr. X. Guo, Pre-general graduate assistant.

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